

Exercise 1

What are the “orders” of the following quantities: $(\mathbf{v} \cdot \mathbf{w})$, $(\mathbf{v} - \mathbf{u})\mathbf{w}$, $(\mathbf{ab} : \mathbf{cd})$, $[\mathbf{v} \cdot \rho\mathbf{wu}]$, $[[\mathbf{a} \times \mathbf{f}] \times [\mathbf{b} \times \mathbf{g}]]$?

Solution

The order of the quantities is determined by adding the orders of the tensors involved (scalar = 0, vector = 1, second-order tensor = 2, third-order tensor = 3, and so on) and subtracting numbers based on what operations are done (dot product = 2, cross product = 1, and double dot product = 4).

$$\begin{aligned}(\mathbf{v} \cdot \mathbf{w}) &\rightarrow \text{Order} = 1 + 1 - 2 = 0 \\(\mathbf{v} - \mathbf{u})\mathbf{w} &\rightarrow \text{Order} = 1 + 1 = 2 \\(\mathbf{ab} : \mathbf{cd}) &\rightarrow \text{Order} = 1 + 1 + 1 + 1 - 4 = 0 \\[\mathbf{v} \cdot \rho\mathbf{wu}] &\rightarrow \text{Order} = 1 + 1 + 1 - 2 = 1 \\[[\mathbf{a} \times \mathbf{f}] \times [\mathbf{b} \times \mathbf{g}]] &\rightarrow \text{Order} = 1 - 1 + 1 - 1 + 1 - 1 + 1 = 1\end{aligned}$$

For the second entry, $\mathbf{v} - \mathbf{u}$ counts as one vector and \mathbf{w} counts as the other.